Conditional preparation of arbitrary superpositions of atomic Dicke states

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We present a protocol capable of preparation of arbitrary superpositions of atomic Dicke states in the form of

\[ |\text{target} \rangle = \sum_{n=0}^{N} c_n |n\rangle. \]

Initial Gaussian light beam in subjected to a squeezing operation followed by conditional single photon subtraction on a beam splitter. Subsequently the light interacts with atomic ensemble via a QND interaction

\[ H_{\text{QND}} = \hbar \kappa \hat{a} \hat{a}^\dagger + \hbar \hat{b} \hat{b}^\dagger. \]

Finally we propose performing homodyne detection of the \( \hat{p}_L \) quadrature of light (conjugate to \( \hat{x}_L \)). Resulting action on atoms reads

\[ \hat{\Theta}(\hat{p}_L) \propto \left[ (\hat{x}_A + \hat{p}_L / \lambda) \exp \left[ -\left( \hat{x}_A + \hat{p}_L / \lambda \right)^2 \right] \right], \]

where atomic quadrature \( \hat{x}_A \) has been defined using collective atomic spin and \( \hat{p}_L \) stands for the outcome of the homodyne detection.

Preparation of superpositions that contain \( |n\rangle \) Dicke state requires \( N \) repetitions of the above described procedure accompanied by application of atomic displacement performed by magnetic field:

\[ \hat{D}(\delta) \hat{a} \]

The best strategy for choosing outcomes of the homodyne detection that will be considered as successful is the usage of fidelity contours as shown in the figure below:

The figure depicts probability distribution (colormap) and final state fidelity (contours) as a function of a pair of possible homodyne detection outcomes \( p_{L1}\) that need to be performed in order to prepare the Dicke state \( |2\rangle \). It is straightforward to see that the optimal choice for the best fidelity - probability trade off is the choice of homodyne detection outcomes satisfying the relation

\[ F(\text{accepted } p_{L1}, p_{L2}) > f_{\text{min}}, \]

where \( f_{\text{min}} \) is some fidelity threshold upon which also the protocol success probability depends.

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