Experimental Implementation of the Optimal Linear-Optical Controlled Phase Gate

Karel Lemr¹, Antonín Černoch¹, Jan Soubusta¹, Konrad Kieling², Jens Eisert²,³ and Miloslav Dušek⁴

¹Joint Laboratory of Optics of Palacký University and Institute of Physics of Czech Academy of Sciences
²Institute of Physics and Astronomy, University of Potsdam
³Institute for Advanced Study Berlin
⁴Department of Optics, Faculty of Science, Palacký University
Why bother?

- Quantum information processing brings new features.
  - More efficient algorithms.
  - Secure communications.
  - True random number generation.
- Building QIP devices is a experimental challenge.
- C-phase gate is an important member of QIP toolbox.
Controlled phase gate

- Controlled phase gate: two qubit gate
  - $|00\rangle \rightarrow |00\rangle$
  - $|01\rangle \rightarrow |01\rangle$
  - $|10\rangle \rightarrow |10\rangle$
  - $|11\rangle \rightarrow e^{i\varphi}|11\rangle$, general phase shift $\varphi$

- Hadamard gate + C-phase gate + Hadamard gate = C-NOT gate
Our implementation

- Using pairs of photons (polarization and spatial modes)
- Gate is probabilistic, performing postselection
- Gate is **optimal for every phase shift**
  - maximum achievable success probability within the framework of linear optics without ancillary photons
- Success probability:
  - \( P_s(\varphi) = \left( 1 + 2 |\sin \frac{\varphi}{2}| + 2^{3/2} \sin \frac{\pi - \varphi}{4} |\sin \frac{\varphi}{2}|^{1/2} \right)^{-2} \)
- **non-monotonoous function of the phase shift**
Experimental setup
Experimental setup

- Type 1 SPDC used as photon pair source
- Setup aligned to perform required phase shift.
- 3 nested Mach-Zehnder interferometers
- Active stabilization necessary
Experimental setup
Measurement

- **Complete process tomography** performed:
  - All combinations of separable input states (H,V,D,A,R,L)
  - Detection of two-photon output state
  - Corresponding process matrix estimated via maximum likelihood method.
Phase shift $\varphi = \pi$

- **Fidelity = 83.5%**
Phase shift $\varphi = \frac{\pi}{2}$

- **Fidelity** = 86.3%
### Results overview

<table>
<thead>
<tr>
<th>$\varphi$ (π)</th>
<th>$F_\chi$</th>
<th>$F_{av}$</th>
<th>$F_{\text{min}}$</th>
<th>$P_{av}$</th>
<th>$P_{\text{min}}$</th>
<th>$p_{s,\text{obs}}$</th>
<th>$p_{s,\text{th}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.939</td>
<td>0.956</td>
<td>0.835</td>
<td>0.960</td>
<td>0.873</td>
<td>0.859 ± 0.013</td>
<td>1.000</td>
</tr>
<tr>
<td>0.050</td>
<td>0.948</td>
<td>0.961</td>
<td>0.906</td>
<td>0.965</td>
<td>0.870</td>
<td>0.366 ± 0.008</td>
<td>0.348</td>
</tr>
<tr>
<td>0.125</td>
<td>0.910</td>
<td>0.903</td>
<td>0.770</td>
<td>0.954</td>
<td>0.866</td>
<td>0.190 ± 0.005</td>
<td>0.210</td>
</tr>
<tr>
<td>0.250</td>
<td>0.842</td>
<td>0.881</td>
<td>0.733</td>
<td>0.896</td>
<td>0.670</td>
<td>0.112 ± 0.003</td>
<td>0.133</td>
</tr>
<tr>
<td>0.500</td>
<td>0.863</td>
<td>0.888</td>
<td>0.815</td>
<td>0.903</td>
<td>0.759</td>
<td>0.090 ± 0.002</td>
<td>0.090</td>
</tr>
<tr>
<td>0.750</td>
<td>0.840</td>
<td>0.868</td>
<td>0.633</td>
<td>0.898</td>
<td>0.705</td>
<td>0.080 ± 0.002</td>
<td>0.088</td>
</tr>
<tr>
<td>1.000</td>
<td>0.835</td>
<td>0.856</td>
<td>0.710</td>
<td>0.922</td>
<td>0.827</td>
<td>0.120 ± 0.001</td>
<td>0.111</td>
</tr>
</tbody>
</table>

- $\varphi$ - phase shift, $F_\chi$ - process fidelity, $F_{av}$ - average state fidelity
- $F_{\text{min}}$ - worst state fidelity, $P_{av}$ - average state purity
- $P_{\text{min}}$ - worst state purity, $p_{s,\text{obs}}$ - observed success probability
- $p_{s,\text{th}}$ - theoretical success probability
Results summary

- Process fidelity: 83% - 95%
- Average purity: 89% - 97%
- errors due to misalignment, polarization dispersion, and active stabilization errors
- measurement for every phase shift took 9 hours
Observed success probability

- Observed success probability verifies theoretical prediction.
- Success probability is a non-monotonous function of $\varphi$. 

![Graph showing success probability as a function of gate phase $\varphi$. The graph plots success probability against gate phase, with data points labeled as 'Theory' and 'Experiment'.]
Conclusions

- implemented optimal controlled phase gate
- good fidelities obtained for all setting of phase shift
- success probability as a function of the phase shift verified
Thank you for your attention